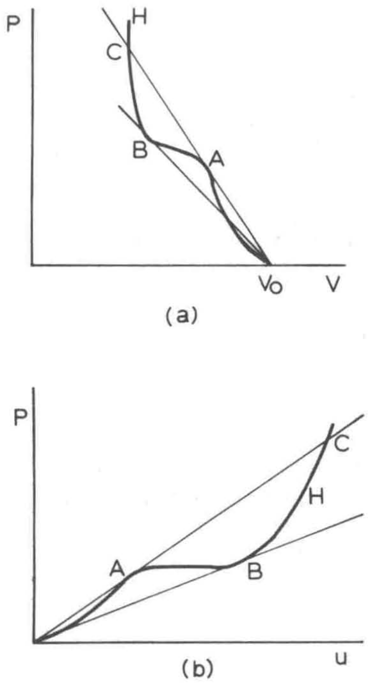

FIG. 19. Hugoniot curve with region of instability between A and C: (a) P-V plane, (b) P-u plane. Point A is entropy maximum along H; B is entropy minimum.

sure of the first wave is stationary at state A with respect to higher shock pressures, (less than C).

Now, consider a situation where the upper limit of Ineq. (19) is exceeded, as illustrated in Fig. 21. In this case a plot of E' as function of P has the appearance shown in Fig. 22. The region between A and B is thermodynamically unstable according to Ineq. (19). Consequently, a shock to state B tends to be stabilized in pressure with respect to lower shock pressures. This is just the situation required for detonation, and we put forward the hypothesis that detonation is indeed the result of a minimum in E' along the Hugoniot curve.

We note that this criterion for detonation is quite different from the Chapman–Jouguet Theory. In that theory detonation corresponds to a local minimum in the entropy along the equilibrium Hugoniot curve. Moreover, it requires the assumption of two effective equations of state, applicable in different regions of the shock transition, a frozen equation of state for the initial shock transition and a relaxed equation of state for the equilibrium state finally achieved (Ref. 10, p. 480).

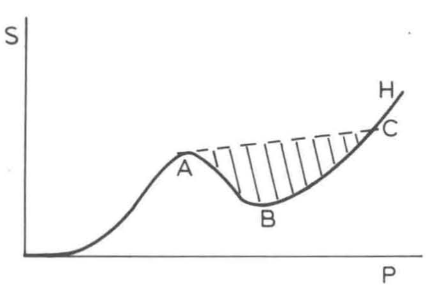

FIG. 20. Entropy as function of pressure along Hugoniot curve of Fig. 19. Cross-hatched region is region of instability.

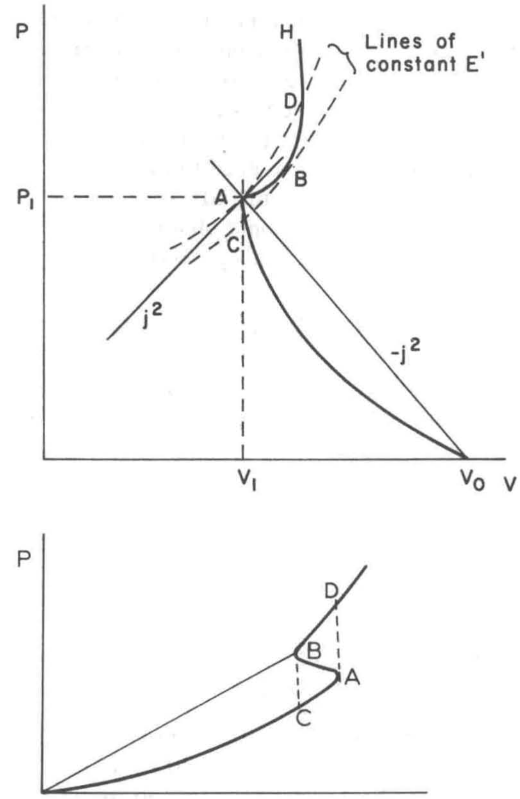

FIG. 21. Hugoniot curve with unstable region $1 < j^2\ (dV/dP)_H$, between A and B: (a) P-V plane, (b) P-u plane. Point A is relative maximum in E' along H; B is relative minimum.

In summary then, stable shocks are characterized by monotonically increasing entropy and reduced internal energy along the Hugoniot curve, and unstable shocks are associated with either a local maximum in the derivative $(dS/dP)_H$, or with a local minimum in the derivative $(dE'/dP)_H$. In the former case, a two-shock configuration results in which the pressure of the first wave is stationary at the entropy maximum. The latter case corresponds to detonation with the pressure stationary at the minimum in E'.

The unstable regions are shown as the cross-hatched areas of Figs. 20 and 22. The upper bound of the unstable region of Fig. 20 is determined by the equivalence of the shock velocity there with that at the entropy maximum. The region CA of Fig. 22 is metastable; shock waves in this range require adequate pertur-

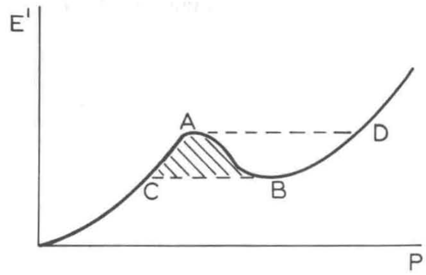

FIG. 22. Plot of E' as function of P along Hugoniot curve of Fig. 21. Cross-hatched region is thermodynamically unstable. Point B is detonation state.

bation to overcome the energy barrier. Just as a liquid can be cooled below the stable transition temperature, the existence of a minimum in E' does not guarantee that a detonation will form; however, state B is the thermodynamically more stable state. It may be for this reason that detonations are observed to propagate more readily in materials that are initially somewhat porous and why turbulence is commonly observed behind detonations. There is an obvious analogy to the onset of turbulence in viscous, steady, subsonic flow.

VI. CONCLUSIONS

We have treated the problem of stability of plane shock waves by considering the reflection of small amplitude acoustic waves from the shock front, and by irreversible thermodynamics. Both approaches yield the same criteria for stability, which can be stated as a restriction on the relative slopes of the Hugoniot curve and the Rayleigh line

$$-1 \leq j^2(dV/dP)_H \leq 1 \ .$$

Violation of the lower limit leads to a two-shock structure; violation of the upper limit to detonation.

The thermodynamic treatment requires the recognition that, at least in an adiabatic mechanical process, the entropy production is bounded above as well as below. This can be stated alternatively by the relations, applicable to real processes,

$$0 \leq TdS \leq (P - P_0)\, dV\,, \quad (T > 0)$$

or by the equivalent relations,

$$0 \leq dS\,; \quad dE' \leq 0\ ,$$

where $dE' = dE + P_0 dV$, is the reduced internal energy.

For shock waves E' is also equal to the kinetic energy density of the shocked state in a coordinate system in which the initial state is stationary; conversely it is the kinetic energy density of the initial state in a coordinate system in which the shocked state is stationary.

Shocks are thermodynamically unstable whenever there is a local maximum in the entropy or a local minimum in the reduced internal energy along the Hugoniot curve. These correspond to a local maximum in the shock velocity and to a local minimum in the particle velocity, respectively. In the former case a two-shock structure develops in which the pressure of the first shock corresponds to the entropy maximum. The latter case gives rise to turbulence that tends to stabilize the shock pressure at the minimum in the reduced internal energy. We posit that detonations are instabilities of this type.

Because E' is also the kinetic energy density, there is another sense in which the stability criteria can be understood. Thus, in a coordinate system fixed in the shocked material the shock front tends to produce maximum entropy with minimum expenditure of the kinetic energy of the incoming material. Instability occurs when there are neighboring Hugoniot states that permit greater production at less cost. It is tempting to speculate that similar thermodynamic conditions may also be valid for biological systems.

We note that the results are in satisfying agreement with a generalized form of the Le Chatelier principle. Thus, for stable shocks both the shock velocity and the particle velocity increase monotonically with pressure.

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[1] H. A. Bethe, Office of Scientific Research and Development, Report 445 (1942).

[2] S. P. D'Yakov, Zh. Eksp. Teor. Fiz. **27**, 288 (1956).

[3] J. J. Erpenbeck, Phys. Fluids **5**, 1181 (1962).

[4] C. Gardner, Phys. Fluids **6**, 1366 (1963).

[5] G. W. Swan and G. R. Fowles, Phys. Fluids **18**, 28 (1975).

[6] G. R. Fowles and G. W. Swan, Phys. Rev. Lett. **30**, 1023 (1973).

[7] L. C. Woods, J. Plasma Phys. **6**, 615 (1971).

[8] R. Courant and K. O. Friedrichs, *Supersonic Flow and Shock Waves* (Wiley, New York, 1948), Chap. 2.

[9] G. R. Fowles, Phys. Fluids **18**, 776 (1975).

[10] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Addison-Wesley, Reading, Mass., 1959).

[11] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Addison-Wesley, Reading, Mass., 1969), p. 99.

[12] G. E. Duvall, *Les Ondes de Detonation* (Editions du Centre National de la Recherche Scientifique, Paris, 1962), p. 337. Also, M. Cowperthwaite, J. Franklin Inst. **285**, 275 (1968).